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Significance of thermophoretic deposition of particles on convective flow between vertical channel in a porous medium with Soret effect

N. Raji Reddy[a,]*, Ch. RamReddy[b]
[a]Department of Mathematics and Humanities, Kakatiya Institute of Technology and Science, Warangal - 505015, Telangana State, India
[b]Department of Mathematics, National Institute of Technology Warangal-506004, Telangana State, India
*Corresponding author Email: narahari.rajireddy@gmail.com

---***ABSTRACT***---
In this paper, the possible solutions for the thermophoretic deposition of particles on a fully developed flow of an electrically conducting fluid saturated porous medium between a vertical channel in the presence of Soret effect, are investigated analyticallyand numerically using shooting method along with 4th Runge-Kutta method. In order to explore the effects of various physical parameters, we also tried to find all possible analytical solutions wherever exist. Finally, the effects of various physical parameters on flow fields are discussed in order to analyze the usefulness of the problem undertaken.

KEYWORDS: *-Thermophoretic Transport; Convective Flow; Vertical Channel; ElectricallyConductingFluid; Porous Medium; Soreteffect.*

I. INTRODUCTION

Transport of heat and mass in a porous medium through a vertical parallel channel, where natural convection plays a significant role in determining the velocity, temperature and concentration fields, have unavoidable practical applications in industrial fields. This geometry, in thermal engineering equipment, is observed to be an often-encountered configuration in collection of solar energy, and cooling devices of electronic and microelectronic equipments (for consolidated literature, one can refer Barletta et al. [1], Nield and Bejan [2] and also the citations therein). The boundary layer flows along a vertical plate in an electrically conducting fluid in the presence of a transverse magnetic field has been discussed by Rossow [3] (also see the citations therein). Since then, several researchers considered unsteady and steady MHD fluid flows under various surface geometries in view of the huge number of applications e.g., to design the cooling system for electronic goods and to obtain many parts in solar energy system, etc. Several researchers [Chamka [4]; Berletta et al. [5]; Pratap kumar et al. [6] etc.] discussed the mixed convective fluid flow problem between parallel vertical plates analytically and mostly numerically.

On the other hand, the importance of Soret effect is noticed in the biological systems, solar pond operation and the world oceans at microstructure level. In particular, due to small thermal gradients, the mass transport across biological membranes in living matter is a key factor in biological systems. Srinivasacharya et al. [7] and RamReddy et al. [8] considered different surface geometries like vertical channel and vertical plate to study the Soret effect with convective boundary layer flow of different fluids (also see the citations therein for more details up to 2013). The thermophoretic effect was first observed by Tyndall [9], by noticing that a particle free zone around a heated surface appeared in dusty air. It also plays an important role in different measurement techniques for aerosols during combustion (Ref. Tsai and Liang [10]). In the papers by Grosan et al. [11] and Magyari [12], numerical and analytical approaches have been reported to the mixed convective flow problem without fluid saturated porous medium in the absence of Soret parameter (Sr) and magnetic parameter (Ha). The aim of the present investigation is to provide the possible closed form solutions for the thermophoretic deposition of particles on a fully developed free convective flow of an electrically conducting fluid with Soret effect in a porous medium. To the best of our knowledge, this problem has not been studied before. Many novel features coming out from this approach are explained in detail.

II. DESCRIPTION OF MATHEMATICAL MODELLING

Consider a steady, laminar, incompressible fully developed flow of an electrically conducting fluid between a vertical channel in a porous medium. A fluid rises in the channel driven by buoyancy forces only. The physical geometry of the problem is displayed in Fig. (1) along with other assumptions. The uniform temperature and concentration at $y=0$ wall are T_h and C_h, and at $y=L$ wall, these are T_c and C_c

respectively, where $T_h > T_c$ and $C_h > C_c$. The applied magnetic field B_0 is uniform and its direction is normal to the fluid flow. The thermophoretic deposition of aerosol particles and Soret effects are included. This analysis further involved the following additional assumptions: (i) the porous medium is homogeneous and isotropic, (ii) the fluid and porous medium properties are kept fixed except in the linear Boussinesq approximation, and (iii) there is local thermodynamic equilibrium between fluid and the porous medium.

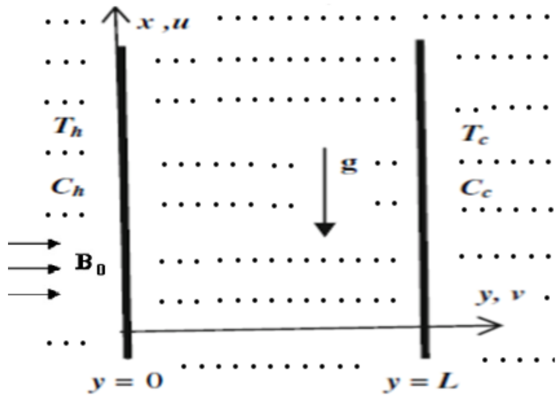


Fig. 1 Schematic diagram and coordinates.

Taking the assumptions into consideration, the momentum, energy and concentration equations for the flow, heat and mass transfer are

$$\mu \frac{d^2u}{dy^2} + \rho g\left[\beta_T\left(T - T_0\right) + \beta_C\left(C - C_0\right)\right] - \frac{\mu\varepsilon}{K}u - \sigma B_0^2 u = \frac{\partial p}{\partial x} \quad (1)$$

$$\frac{d^2T}{dy^2} = 0 \quad (2)$$

$$D\frac{d^2C}{dy^2} + \left(\frac{DK_T}{T_M} + k\,\nu C\right)\frac{d^2T}{dy^2} + \frac{k\,\nu}{T}\frac{dC}{dy} = 0 \quad (3)$$

along with the boundary conditions

$$u(0) = u(L) = 0,\, T(0) = T_h,\, T(L) = T_c,\, C(0) = C_h,\, C(L) = C_c \quad (4)$$

where g is the acceleration due to gravity, K is the (intrinsic) permeability of the medium, p is the dynamic pressure, D is the diffusion coefficient, μ is the dynamic viscosity, β_T and β_C are the thermal andconcentration expansion coefficients, ρ being the fluid density, ε is the porosity, k is the non-dimensional thermophoretic coefficient [Talbot *et al.* (1980)], ν is the kinematic viscosity, $T_M$is the mean fluid temperature, K_T is the thermal diffusion ratio, $T_0 = \left(T_h + T_c\right)/2$ and $C_0 = \left(C_h + C_c\right)/2$ are the characteristic temperature and concentration respectively.

As there is no external pressure force in the case of free convection heat and mass transfer. Therefore, we have $\frac{\partial p}{\partial x} = 0$ in Eq. (1).

In order to solve Eqs.(1) – (3) along with the B.Cs.(4), these dimensionless variables are used:

$$Y = \frac{y}{L},\, U(Y) = \frac{uL\rho}{Gr\mu},\, \theta(Y) = \frac{T - T_0}{T_h - T_c},\, \phi(Y) = \frac{C - C_0}{C_h - C_c},\quad V_T = -\frac{k}{\nu\left(Nt + \theta\right)}\frac{d\theta}{dY} \quad (5)$$

where $Nt = (T_h + T_c)/(T_h - T_c)$ is a non-dimensional parameter. Using the transformations (5) into Eqs. (1) – (3), we obtained the following system of ordinary differential equations:

$$\frac{d^2U}{dY^2}+\left(\theta+B\phi\right)-Ha^2U-\frac{\varepsilon}{Da}U=0 \quad (6)$$

$$\frac{1}{Pr}\frac{d^2\theta}{dY^2}=0 \quad (7)$$

$$\frac{1}{Sc}\frac{d^2\phi}{dY^2}+Sr\frac{d^2\theta}{dY^2}+k\frac{d}{dY}\left[\frac{\left(\phi+Nc\right)}{\left(Nt+\theta\right)}\frac{d\theta}{dY}\right]=0 \quad (8)$$

The B.Cs. (4) reduce to

$$U(0)=U(1)=0,\ \theta(0)=1/2,\ \theta(1)=-1/2,\ \phi(0)=1/2,\ \phi(1)=-1/2 \quad (9)$$

where $N_c=(C_h+C_c)/(C_h-C_c)$ is the non-dimensional parameter, $Ha^2=\sigma B_0^2L^2/\left(\upsilon\rho\right)$ is the magnetic parameter, $\Pr=\upsilon/\alpha$ is the Prandtl number, $Sc=\upsilon/D$ is the Schmidt number and $Da=k/L^2$ is the Darcy parameter. Finally, $B=\beta_C(C_h-C_c)/\left(\beta_T(T_h-T_c)\right)$ is the buoyancy parameter and $Sr=DK_T\left(T_h-T_c\right)/\left(T_m\upsilon\left(C_h-C_c\right)\right)$ is the Soret parameter.

The physical quantities of interest in this problem are the shear stress, heat and mass transfers from the plate are given by

$$\tau_x=-\mu\left(\frac{\partial u}{\partial y}\right)_{y=0,1},\ q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0,\,1},\quad q_m=-D\left(\frac{\partial C}{\partial y}\right)_{y=0,\,1} \quad (10)$$

The skin friction coefficient, Nusselt number and Sherwood number, which are defined as

$$C_f=\left(\frac{dU}{dY}\right)_{Y=0,1},\quad \mathrm{Nu}=\left(\frac{d\theta}{dY}\right)_{Y=0,1},\quad \text{and}\quad \mathrm{Sh}=\left(\frac{d\phi}{dY}\right)_{Y=0,1} \quad (11)$$

III. POSSIBLE ANALYTICAL SOLUTIONS

In this section, we tried to investigate the possible analytical solutions. Hence, we noticed the following possibilities for the solutions analytically.

Case(a): When k = 0, the present problem becomes the fully developed MHD convective flow in a porous medium between vertical channel in the absence of thermophoretic particle deposition. In this case, the dimensionless velocity, temperature and concentration profiles are obtained analytically from Eq. (6) - Eq. (8) subjected to the boundary conditions (9) as given below:

$$\left.\begin{aligned}&U(Y)=\frac{(1+e^{-M})}{2M^2}\frac{\left(1+B\right)}{\left(e^{-M}-e^{M}\right)}e^{-MY}+\frac{(1+e^{M})}{2M^2}\frac{(1+B)}{(e^{-M}-e^{M})}e^{MY}+\left(\frac{1+B}{M^2}\right)\left(\frac{1}{2}-Y\right)\\&\theta(Y)=-Y+\frac{1}{2}\quad\text{and}\quad\phi(Y)=-Y+\frac{1}{2}\end{aligned}\right\} \quad (12)$$

where $M^2=Ha^2+\dfrac{\varepsilon}{Da}$ is a constant.

Case(b): When $k\neq 0$, the present problem becomes the fully developed MHD convective flow in a porous medium between vertical channel in the presence of thermophoretic particle deposition. On integrating Eq. (7), we get the solution of the energy equation as given below

$$\theta=-Y+\frac{1}{2} \quad (13)$$

This problem plays a vital role in obtaining solution for ϕ from Eq. (8) as it is involved in Eq. (6) openly (in addition to velocity and θ). For various values of involved parameters, Grosan*et al.* (2009) and Magyari (2009) gave possible implicit and explicit solutions in the case of mixed convection problem without Soret parameter. In the presence of Soret effect for free convective flows, we proceed as shown below to find out explicit representation for ϕ from Eq. (8)

$$\frac{d\phi}{dY} = -k\ Sc\ \frac{Nc+\phi}{Nt+\theta}\ \frac{d\theta}{dY} + C_1 = k\ Sc\ \frac{Nc+\phi}{\left(Nt+\frac{1}{2}\right)-Y} + C_1 \qquad (14)$$

where C_1 is a integration constant. The introduction of new independent variable Z where $Z = \left(Nt + \frac{1}{2} - Y\right)$ and the notation a given by equation $a = k\,Sc$ makes equation (14) as

$$\frac{d\phi}{dZ} = -\frac{a\left(Nc+\phi\right)}{Z} - C_1 \qquad (15)$$

It is important to note that $a = k\,Sc$ highlights the materially significant feature, that is there is no dependency of the k (thermophoretic coefficient) and Sc (Schmidt number) unconnectedly on the boundary value problem (6) - (8) along with conditions (9) but it is on their product $k\ Sc$ only. So, it is obvious that this principle of equivalent states is a consequence of the interplay between diffusion (driven by concentration gradients) and thermophoresis (driven by temperature gradients).

Fortunately, Eq. (15) can be integrated once more, yielding the explicit analytical solution

$$\phi(Y) = -Nc - \frac{C_1}{(a+1)} Z + \frac{C_2}{Z^a} \qquad (16)$$

where C_2 is the second integration constant of Eq. (8). Considering ϕ and B.Cs. (9), the expressions for C_1 and C_2 are obtained as

$$C_1 = \frac{(a+1)\left[C_2\left(Nt+\frac{1}{2}\right)^{-a} - \left(Nc+\frac{1}{2}\right)\right]}{\left(Nt+\frac{1}{2}\right)},$$

$$C_2 = \frac{\left[\left(Nc+\frac{1}{2}\right)\left(Nt+\frac{1}{2}\right) - \left(Nc-\frac{1}{2}\right)\left(Nt+\frac{1}{2}\right)\right]\left(Nt^2-\frac{1}{2}\right)^a}{\left(Nt-\frac{1}{2}\right)^{a+1} - \left(Nt+\frac{1}{2}\right)^{a+1}}$$

Accordingly,

$$\phi = -Nc + \frac{(2Nc+1)(2Nt+1)^a - (2Nc-1)(2Nt-1)^a}{(2Nt+1)^{a+1} - (2Nt-1)^{a+1}}\ Z + \frac{2^{1-a}(Nc-Nt)\left(4Nt^2-1\right)^a}{(2Nt+1)^{a+1} - (2Nt-1)^{a+1}}\ \frac{1}{Z^a} \qquad (17)$$

Thus, an explicit form of the analytical solution of the ϕ - problem is also available. It is important to observe that, in view of the respective expression for Nt and Nc, for the allowed values of Nc and Nt, the variations $Nc < \frac{1}{2}$ *and* $Nc > 1/2$ hold which is sufficient to say that no imaginary quantities and no singularities can appear in Eq. (17).

To consider velocity problem, Eq. (13) and Eq. (16) are substituted in Eq. (6) and the second order non-homogeneous ordinary differential equation is obtained as

$$\frac{d^2U}{dY^2} + \left(-Y + \frac{1}{2} + B\left(-Nc - \frac{C_1 Z}{(a+1)} + \frac{C_2}{Z^a}\right)\right) - Ha^2 U - \frac{\varepsilon}{Da} U = 0 \qquad (18)$$

The general solution of Eq. (18) is given by

$$U(Y) = C_3 e^{-MY} + C_4 e^{MY} - \frac{1}{M^2}\left[\left\{Nt - Z - B\left(-Nc - \frac{C_1 Z}{(a+1)} + \frac{C_2}{Z^a}\right)\right\} + \left(\frac{D^2}{M^2} + \frac{D^4}{M^4} + ...\right)\left(\frac{-BC_2}{Z^a}\right)\right] \qquad (19)$$

where $M^2 = Ha^2 + \frac{\varepsilon}{Da}$.

Therefore, the analytical closed form solution of Eqs.(6) - (8)

(i) will not exist when a = -1.

(ii) U will become a non-terminating series when a = any positive integer or fraction.

(iii) will exist as follows when a = any negative integer smaller than -1.

If a = -2, then the analytical solution of Eq. (19) is

$$U(Y) = C_3 e^{-MY} + C_4 e^{MY} - \frac{1}{M^2}\left[\left\{Nt - Z - B\left(-Nc + C_1 Z + C_2 Z^2\right)\right\} - \frac{2BC_2}{M^2}\right]$$

If a = -3, then the analytical solution of Eq. (19) is

$$U(Y) = C_3 e^{-MY} + C_4 e^{MY} - \frac{1}{M^2}\left[\left\{Nt + Z + B\left(-Nc + \frac{C_1 Z}{2} + C_2 Z^3\right)\right\} - \frac{6BC_2}{M^2} Z\right]$$

and so on.

Therefore, the analytical closed form solution of Eqs. (6) - (8) (when a = -2) is given by

$$\left.\begin{aligned} &U(Y) = C_3 e^{-MY} + C_4 e^{MY} - \frac{1}{M^2}\left[\left\{Nt - Z - B\left(-Nc + C_1 Z + C_2 Z^2\right)\right\} - \frac{2BC_2}{M^2}\right] \\ &\theta(Y) = -Y + \frac{1}{2} \text{ and } \phi(Y) = -Nc - \frac{C_1 Z}{(a+1)} + \frac{C_2}{Z^a} \end{aligned}\right\} \quad (20)$$

Using the MATHEMATICA software along with the boundary conditions (9), we obtain the values of C_3 and C_4. For sake of brevity, the values of C_3 and C_4 are not given here. However, the values of C_1 and C_2 are given below

$$C_1 = \left(Nt - \frac{1}{2}\right)\left[\frac{\left(Nc + \frac{1}{2}\right)}{\left(Nt + \frac{1}{2}\right)} - C_2\right],$$

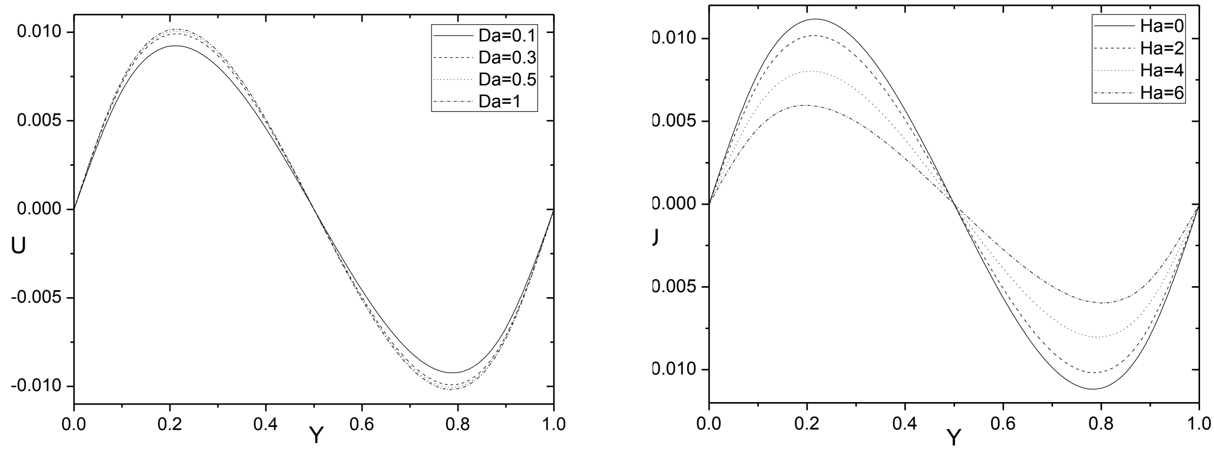
$$C_2 = \left[\frac{\left(Nt - \frac{1}{2}\right)}{\left(Nt + \frac{1}{2}\right)} + \left(\frac{1}{2} + Nc\right) + \left(\frac{1}{2} + Nc\right)\right]\left[\frac{\left(Nt - \frac{1}{2}\right)^{-2}\left(Nt + \frac{1}{2}\right)^{-1}}{\left(Nt - \frac{1}{2}\right)^{-1} - \left(Nt + \frac{1}{2}\right)^{-1}}\right]$$

There is no need of any extra comment between the linear distribution of temperature and the channel subject to the isothermal walls because it is a routine textbook matter. Regarding the concentration solution provided in (20), firstly it is noticed that it depends on the transformed transverse coordinate $Z = \left(Nt + \frac{1}{2} - Y\right)$ in a pure algebraic way, including a strongly nonlinear term proportional to $\frac{1}{Z^a}$ in addition to the linear term in Z. The explicit expression (20) of ϕ contains parameters Nt, Nc and $a = k\ Sc$ only so it is important to note its independency from Darcy parameter (Da), Magnetic parameter (Ha) and buoyancy ratio B.

IV. RESULTS AND DISCUSSION

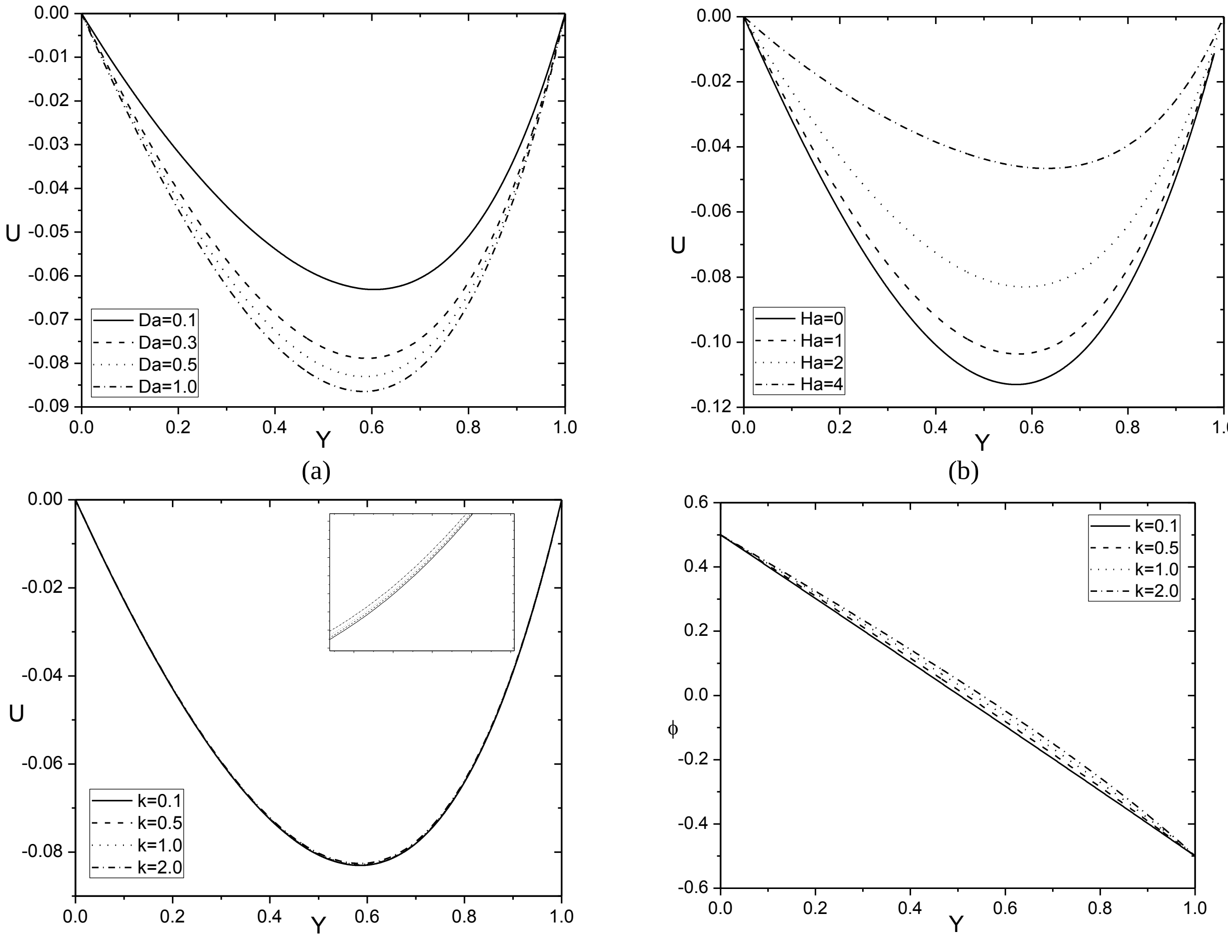
The non-dimensional system of differential equations is solved analytically and numericallyusing shooting method along with 4th Runge-Kutta method for the dimensionless temperature θ, concentration ϕ and velocity U, respectively. This section aims to explain few basic and important properties of these solutions.

The Computations are done when $B = 0.5,\ Sc = 0.5,\ \varepsilon = 0.6,\ Nc = 0.5,\ Nt = 1.0,\ \Pr = 0.71$. These values are fixed throughout the work unless otherwise mentioned.



(a) (b)

Fig.2: *Effect of (a) Darcy parameter and (b) magnetic parameter on velocity profile.*



(a) (b)

(c) (d)

Fig.3: *Effect of (a) Darcy parameter, (b) magnetic parameter, (c) thermophoretic parameter on velocity profile and (d) thermophoretic parameter on concentration profile.*

Figures 2(a) – 2(b) are displayed to illustrate the effects of Darcy parameter (*Da*) and magnetic parameter (*Ha*) on velocity profiles for *k=0* (in the absence of thermophoretic particle deposition effect). Fig.2(a) indicates that, the increasing values of Darcy parameter (*Da*) increases the velocity in the left half of the channel but the reverse trend can be noticed in the right half of the channel. Further, it is noticed from Fig.2(b) that the enhancement of magnetic parameter (*Ha*) reduces the velocity in the left half of the channel and the reverse trend can be noticed in the right half of the channel. The effects of Darcy parameter (*Da*), magnetic parameter (*Ha*), thermophoretic parameter (*k*) on velocity profiles and thermophoretic parameter (*k*) on concentration profiles are shown in Figs.3(a) – 3(d). It is noticed that the enhancement of magnetic parameter (*Ha*) and thermophoretic parameter (*k*) increases the velocity whereas Darcy parameter (Da) decreases the velocity. Figure 3(d) depicts that by increasing the thermophoretic parameter (*k*), velocity increases in the flow field. Table (1) is prepared to display the effects of Darcy parameter and magnetic parameter on skin friction coefficients at left and right walls of the channel. From all the values, it is noticed that the skin friction increases with the increase of *Da* whereas showing reverse trend with the increase of *Ha* at both the plates.

Table 1: *Values of skin friction coefficient for varying values of MHD and Darcy parameter effects.*

k	Ha	Da	$\left(\frac{dU}{dY}\right)_{Y=0}$	$\left(\frac{dU}{dY}\right)_{Y=1}$
0.0	0.0	1.0	0.123766	0.123766
0.0	1.0	1.0	0.121788	0.121788
0.0	2.0	1.0	0.116357	0.116357
0.0	2.0	0.1	0.108136	0.108136
0.0	2.0	0.3	0.114052	0.114052
0.0	2.0	0.5	0.115353	0.115353
0.5	0.0	1.0	0.126357	0.120703
0.5	1.0	1.0	0.124140	0.118970
0.5	2.0	1.0	0.118206	0.114077
0.5	2.0	0.1	0.109462	0.106422
0.5	2.0	0.3	0.115730	0.111954
0.5	2.0	0.5	0.117124	0.113155

V. CONCLUSION

In this study, the impacts of MHD, deposition of thermophoretic particles and Soret parameter on fully developed free convective flow between vertical channel in a porous medium, have been analysed analytically and numerically. The important conclusions of this work can be summarized as:

- It is observed that the velocity and concentration increase with the thermophoretic parameter.
- The velocity increases/decreases in the left half of the channel and diminishes/augments in the right half of the channel with increasing values of Darcy parameter/magnetic parameter respectively in the absence of thermophoretic parameter but the velocity decreases in the presence of thermophoretic parameter.
- It is found the Soret parameter has no significance role in the present study.

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